Attendance Problems.

1. What is the third angle measure in a triangle with angles measuring 65° and 43°?

Find each value. Round trigonometric ratios to the nearest hundredth and angle measures to the nearest degree.

2. $\sin 73°$ **3.** $\cos 18°$ **4.** $\tan 82°$

5. $\sin^{-1}(0.34)$ **6.** $\cos^{-1}(0.63)$ **7.** $\tan^{-1}(2.75)$

I can use the Law of Sines and the Law of Cosines to solve triangles.

<u>Common Core</u>

CC.9-12.G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.
CC.9-12.G.SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and nonright triangles
CC.9-12.G.SRT.10 (+) Prove the Laws of Sines and Cosines and use them to solve problems.
CC.9-12.G.SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and nonright triangles.

In this lesson, you will learn to solve *any* triangle. To do so, you will need to calculate trigonometric ratios for angle measures up to 180°. You can use a calculator to find these values.

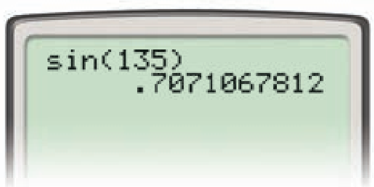
<u>Video Example 1</u>. Use your calculator to find each trigonometric ratio. Round to the nearest hundredth.

A. $\sin 127°$ **B.** $\tan 54°$ **C.** $\cos 101°$

1 Finding Trigonometric Ratios for Obtuse Angles

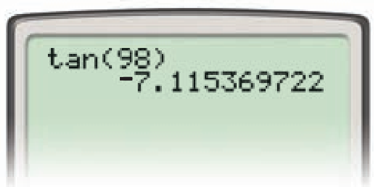
Use a calculator to find each trigonometric ratio. Round to the nearest hundredth.

A $\sin 135°$



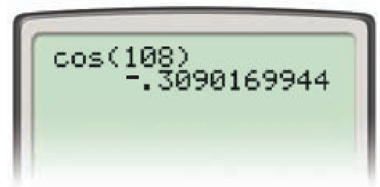
$\sin 135° \approx 0.71$

B $\tan 98°$



$\tan 98° \approx -7.12$

C $\cos 108°$



$\cos 108° \approx -0.31$

Example 1. Use your calculator to find each trigonometric ratio. Round to the nearest hundredth.

A. tan 103° **B.** cos 165° **C.** sin 93°

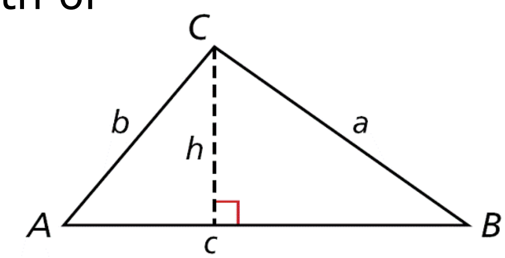
<u>Guided Practice</u>. Use your calculator to find each trigonometric ratio. Round to the nearest hundredth.

8. $\tan 175°$ **9.** $\cos 92°$ **10.** $\sin 160°$

You can use the altitude of a triangle to find a relationship between the triangle's side lengths.

In ΔABC, let h represent the length of the altitude from C to $\overline{AB}$.

From the diagram, $\sin A = \frac{h}{b}$, and $\sin B = \frac{h}{a}$.



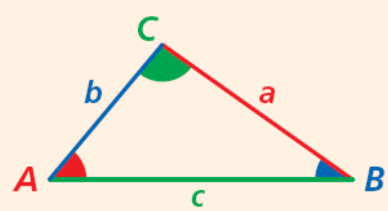
By solving for h, you find that $h = b \sin A$ and $h = a \sin B$. So $b \sin A = a \sin B$, and $\frac{\sin A}{a} = \frac{\sin B}{b}$.

You can use another altitude to show that these ratios equal $\frac{\sin C}{c}$.

Theorem 8-5-1 The Law of Sines

For any $\triangle ABC$ with side lengths a, b, and c,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

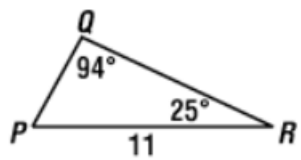


You can use the Law of Sines to solve a triangle if you are given

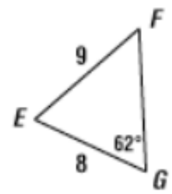
- two angle measures and any side length (ASA or AAS) or
- two side lengths and a non-included angle measure (SSA).

<u>Video Example 2</u>. Find the measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

A. PQ



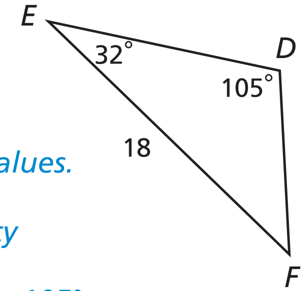
B. $m\measuredangle F$



2 Using the Law of Sines

Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

A DF



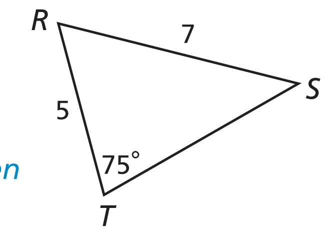
$$\frac{\sin D}{EF} = \frac{\sin E}{DF}$$ *Law of Sines*

$$\frac{\sin 105°}{18} = \frac{\sin 32°}{DF}$$ *Substitute the given values.*

$$DF \sin 105° = 18 \sin 32°$$ *Cross Products Property*

$$DF = \frac{18 \sin 32°}{\sin 105°} \approx 9.9$$ *Divide both sides by sin 105°.*

B $m\angle S$



$$\frac{\sin T}{RS} = \frac{\sin S}{RT}$$ *Law of Sines*

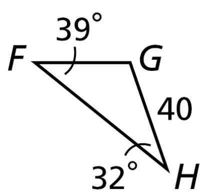
$$\frac{\sin 75°}{7} = \frac{\sin S}{5}$$ *Substitute the given values.*

$$\sin S = \frac{5 \sin 75°}{7}$$ *Multiply both sides by 5.*

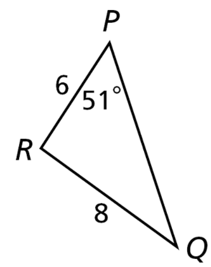
$$m\angle S \approx \sin^{-1}\left(\frac{5 \sin 75°}{7}\right) \approx 44°$$ *Use the inverse sine function to find m∠S.*

<u>Example 2.</u> Solve the triangle. Round lengths to the nearest tenth and angle measures to the nearest degree.

A.

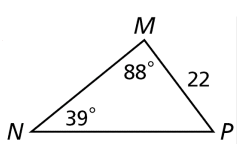


B.

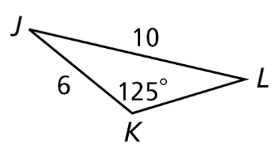


<u>Guided Practice</u>. Solve the triangle. Round lengths to the nearest tenth and angle measures to the nearest degree.

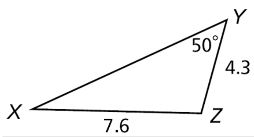
11. NP



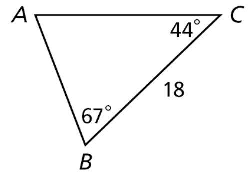
12. YZ



13. $m\measuredangle X$



14. AC

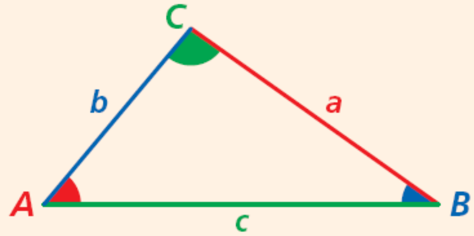


The Law of Sines cannot be used to solve every triangle. If you know two side lengths and the included angle measure (SAS) or if you know all three side lengths (SSS), you cannot use the Law of Sines. Instead, you can apply the Law of Cosines.

Theorem 8-5-2 **The Law of Cosines**

For any $\triangle ABC$ with side lengths a, b, and c:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

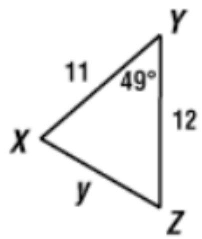


Helpful Hint

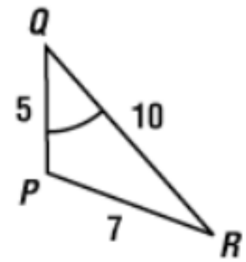
The angle referenced in the Law of Cosines is across the equal sign from its corresponding side.

<u>Video Example 3</u>. Find the measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

A. XZ



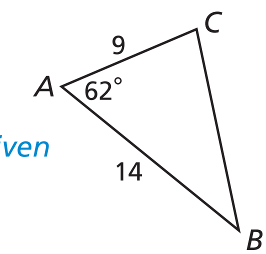
B. $m\measuredangle Q$



3 Using the Law of Cosines

Find each measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

A BC



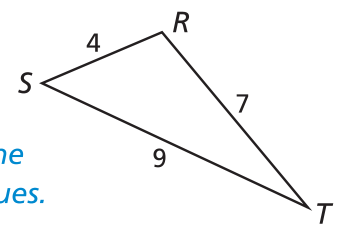
$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos A$ — *Law of Cosines*

$= 14^2 + 9^2 - 2(14)(9)\cos 62°$ — *Substitute the given values.*

$BC^2 \approx 158.6932$ — *Simplify.*

$BC \approx 12.6$ — *Find the square root of both sides.*

B $m\angle R$



$ST^2 = RS^2 + RT^2 - 2(RS)(RT)\cos R$ — *Law of Cosines*

$9^2 = 4^2 + 7^2 - 2(4)(7)\cos R$ — *Substitute the given values.*

$81 = 65 - 56\cos R$ — *Simplify.*

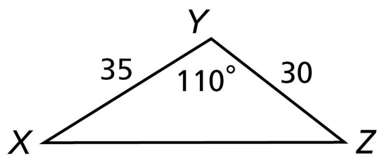
$16 = -56\cos R$ — *Subtract 65 from both sides.*

$\cos R = -\frac{16}{56}$ — *Solve for cos R.*

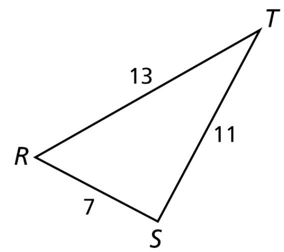
$m\angle R = \cos^{-1}\left(-\frac{16}{56}\right) \approx 107°$ — *Use the inverse cosine function to find m∠R.*

Example 3. Solve the triangle. Round lengths to the nearest tenth and angle measures to the nearest degree.

A.

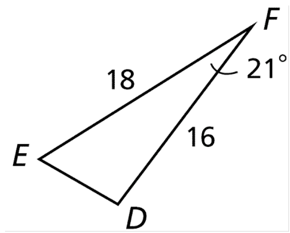


B.

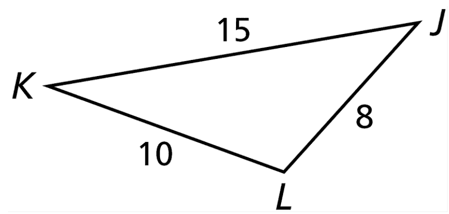


<u>Guided Practice</u>. Find the measure. Round lengths to the nearest tenth and angle measures to the nearest degree.

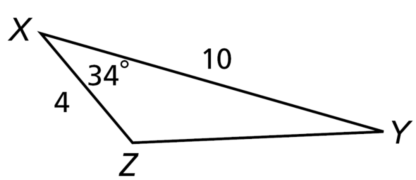
15. DE



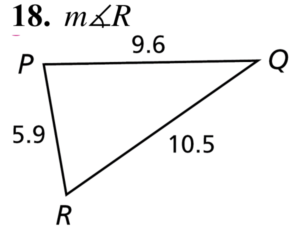
16. $m\measuredangle K$



17. YZ



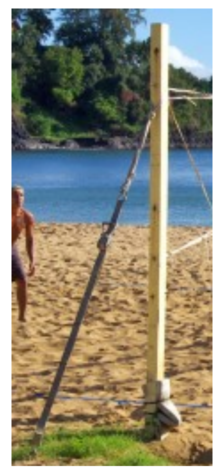
18. $m\measuredangle R$



Helpful Hint

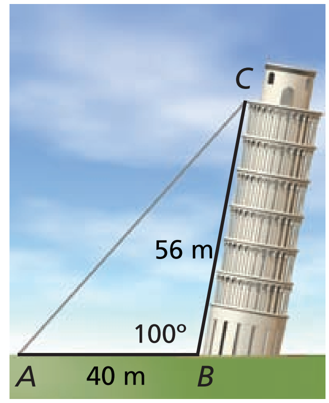
Do not round your answer until the final step of the computation. If a problem has multiple steps, store the calculated answers to each part in your calculator.

Example 4. The pole that supports a beach volleyball net is 2.25 meters tall and makes a $91°$ angle with the ground. To stabilize the nest, a rope is attached the top pole and extends to the ground 1.75 meters from the base of the pole. How long is the rope, and what angle does it make with ground? Round the length to the nearest tenth and the angle measures to the nearest degree.



4 Engineering Application

The Leaning Tower of Pisa is 56 m tall. In 1999, the tower made a 100° angle with the ground. To stabilize the tower, an engineer considered attaching a cable from the top of the tower to a point that is 40 m from the base. How long would the cable be, and what angle would it make with the ground? Round the length to the nearest tenth and the angle measure to the nearest degree.



Step 1 Find the length of the cable.

$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos B$ — *Law of Cosines*

$= 40^2 + 56^2 - 2(40)(56)\cos 100°$ — *Substitute the given values.*

$AC^2 \approx 5513.9438$ — *Simplify.*

$AC \approx 74.3$ m — *Find the square root of both sides.*

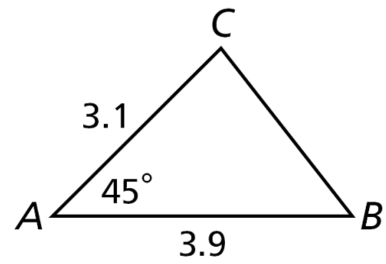
Step 2 Find the measure of the angle the cable would make with the ground.

$\frac{\sin A}{BC} = \frac{\sin B}{AC}$ — *Law of Sines*

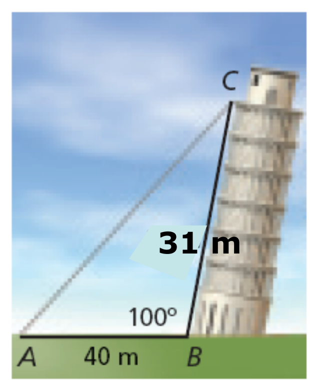
$\frac{\sin A}{56} \approx \frac{\sin 100°}{74.2559}$ — *Substitute the calculated value for AC.*

$\sin A \approx \frac{56 \sin 100°}{74.2559}$ — *Multiply both sides by 56.*

$m\angle A \approx \sin^{-1}\left(\frac{56 \sin 100°}{74.2559}\right) \approx 48°$ — *Use the inverse sine function to find m∠A.*



Example 4. A sailing club has planned a triangular racecourse, as shown in the diagram. How long is the leg of the race along *BC*? How many degrees must competitors turn at point *C*? Round the length to the nearest tenth and the angle measure to the nearest degree.



19. Guided Practice. Another engineer suggested using a cable attached from the top of the tower to a point 31 m from the base. How long would this cable be, and what angle would it make with the ground? Round the length to the nearest tenth and the angle measure to the nearest degree.

8-5 Law of Sines and Cosines (pp 573) 23, 25, 29, 31, 35, 37, 38, 42, 46, 49, 50-54, 56, 58.